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ON SPATIAL UNCERTAINTY IN A SURFACE LONG BASELINE POSITIONING SYSTEM

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This paper describes a general method for estimating the probability distribution of an acoustic source resulting from noisy measurements in a surface long baseline (SLBL) positioning system. Compared with Monte Carlo simulations this method calculates the mean and covariance at a reduced computational cost, due to a deterministic sampling strategy. Hence, it can easily be checked if a SLBL configuration is well suited to a particular positioning scenario.

1. INTRODUCTION

High precision positioning is of particular interest in various underwater application areas (marine salvage, offshore survey, mineral exploration, seafloor mapping, ROV tracking...). Numerous sophisticated systems are used for this purpose today but no global solution allowing performances comparable to GPS [1] is available yet. With traditional long baseline (LBL) acoustic positioning systems, a centimeter-level accuracy is sometimes attainable but the time required to deploy and calibrate the transponders array on the seabed become a severe drawback for some applications. Short baseline (SBL) and ultra short baseline (USBL) systems only require an acoustic unit composed of multiple hydrophones, mounted on a survey ship, but suffer from loss of accuracy in shallow water.

The idea to use GPS technology underwater has been a major advance over the past ten years. The first solution, due to Youngberg from the US-Air Force, is a direct transposition of the GPS technique to underwater world: satellites are replaced by surface buoys and the mobile, equipped with a receiver, processes location by measuring time of arrival (TOA) of acoustic pulses transmitted by the buoys. A reverse configuration has been proposed later by Thomas [1]: the principle is based on measuring on a set of buoys, the TOA of an acoustic pulse sent by the vehicle. At regular intervals, each buoy transmits to a survey ship its GPS position and the TOAs of the acoustic pulses. Assuming the sound velocity in seawater is known, the distances lead to the vehicle's location. Throughout this paper we will refer to this method as **surface long baseline** (SLBL) system. SLBL constitutes a very attractive positioning solution: it is easily deployable, and it can cover a wide operation area with meter-level accuracy.

Obviously, assumptions such as known sensor locations, good quality measurements or well-known vehicle dynamic are not realistic. Due to complex and non-stationary characteristics of the acoustic environment and variable atmospheric conditions, it is necessary to reason on the basis of inaccurate information. As pointed out by Sutherland et al. [3] in a different context, the combination of approximate measures can lead to large errors in localization, even if robust tracking algorithms are applied. These localization errors can be greatly reduced by a good choice of the location of sensors [5].

In this paper, we describe a general method for estimating the spatial uncertainty in vehicle's localization. Our approach is based on a transformation proposed by Julier et al. [4] for calculating the statistics of a random variable (r.v.) which undergoes a non-linear transformation. Thanks to this method, the mean and covariance of the probability density function (PDF) of the vehicle's location are estimated in an efficient manner. The explicit form of the non-linear functions is not required and the first two moments of the PDF are precisely captured at a reduced computational cost in comparison with Monte Carlo simulations. This permits to decide in advance whether a particular SLBL configuration is able to provide good localization regardless of measurement errors.

The paper is organized as follows. In section 1 we briefly describe the principle of a SLBL. Section 2 examines various causes of error in such a system and how they affect errors in localization. Section 3 presents the Julier procedure to calculate the statistics of a r.v. obtained through a non linear function. Then, in section 4, we apply this method to model the uncertainty resulting from inexact information when using a SLBL. An illustrative example is proposed in section 5 with a towed sonar tracking scenario. Concluding remarks are given in section 6.

2. SURFACE LONG BASELINE POSITIONING

The SLBL is a recent concept [2] which uses the strong potential of the GPS while taking into account the principle of the LBL. The vehicle to be localized is equipped with a precision clock, synchronous to the GPS time. In order to avoid interrogation-response cycles usually utilized in such systems, the target transmits a signal at preprogrammed instants. Accordingly the sources of errors are decreased and the sampling rate is increased. The buoys can be anchor on the operation area or can be let drifting according to working conditions. Each buoy has to: detect the TOA of the acoustic signal transmitted by the target, date precisely this TOA using the clock reference of the GPS, update its own position using the GPS and transmit the collected information towards a central computer (by VHF in general).

The central computer determines the target coordinates at the transmission time using the whole collected data. Usually, this determination can be realized by hyperbolic or pseudo-circular algorithms. In this paper the pseudo-circular mode has been considered.

To improve its localization, the mobile can transmit its depth. This additional information improves the precision of the system and permits to reduce the number of unknown parameters to two (longitude and latitude). In order to keep simplicity and clearness figures are presented in 2D.

The main advantage of the SLBL, which can be assimilated to a reverse LBL, lies in its high precision without calibration even in long range operation. Moreover, the sensors are located far from any source of noise (except the surface of the sea) and then allow low level transmitted signals. These systems conceived by ACSA are manufactured and marketed in France by ORCA instrumentation. They allow a high rate of sampling in trajectory calculation.

3. THE EFFECTS OF INACCURATE INFORMATION

Assuming a two dimensional space, a straightforward analysis of the error-free SLBL localization problem yields the acoustic source location: knowledge of the sound speed and various TOA gives the distance of the source with respect to each buoy, and then a set of circles centered on the buoys intersect at a single point. Unfortunately, we must deal with noisy measurements and then the true location lies somewhere in an *area of uncertainty*, whose size and shape depend upon error amount and buoys positions. As noticed by Sutherland [3] for a robot navigating in an unstructured outdoor environment, it is crucial to precisely estimate this uncertainty.

The following errors will be encountered in a SLBL positioning system:

- the sound velocity used for calculation is an approximate value;
- as the signals of GPS satellites are corrupted by various errors (data noise, multipath errors, clock errors, atmospheric delays,…) the buoys locations are uncertain;
- the hydrophones are not precisely located with respect to the buoys because of sea state fluctuations;
- estimation of the TOA is subject to environmental noises (reflections, multipath,…).

In order to characterize the spatial uncertainty resulting from these inexact information, a least mean square (LMS) criteria will be employed in the following : given the set of circles coming from the measured distances between the acoustic source and the buoys, the searched position will correspond to the point which minimizes the sum of the squared perpendicular distances to the circles.

Then, the statistics of the source location can be explored by means of Monte Carlo simulations. This approach is illustrated in Fig. 1, which corresponds to 10000 trials, assuming Gaussian error models for the three buoys positions, the sound speed and the TOA. Supposing that the resulting probability distribution is Gaussian, the 95 % confidence ellipse is drawn to reveal the spatial uncertainty of the source location.

Now we can ask if a better localization could be achieved with more than three buoys. Numerous simulations have been carried out to answer this question : we can note that the spatial uncertainty does not necessarily decrease by adding buoys, and in most cases a good three buoys-configuration would perform better than a configuration having more buoys. This characteristic is clearly revealed by the second simulation (Fig. 2), performed with a different configuration of four buoys (same error models):

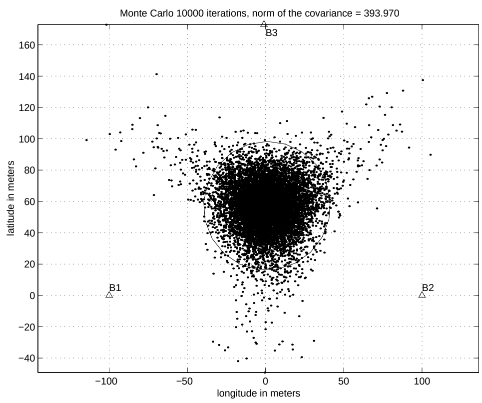


Fig. 1: MonteCarlo simulation / 3 buoys

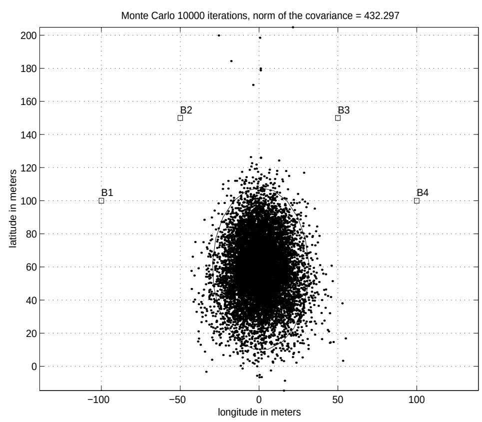


Fig. 2: Monte Carlo simulation / 4 buoys

We can observe that the first configuration (3 buoys) is less sensitive to measurement errors than the last one.

The purpose of this paper is to propose an efficient method to estimate the spatial uncertainty in order to find a configuration well suited to the desired positioning scenario.

In the following, the acoustic source location will be considered as a r.v. resulting from the non linear transformation of the r.v. associated to uncertain environmental parameters and noisy measurements (sound celerity, buoys location, TOA,…). The mean and covariance of the source probability density function (pdf) will be calculated through a general method recently developed by Julier et al. [4] for non linear estimation problems. This method avoids the large computational cost usually encountered with Monte Carlo simulation technique, and is more accurate than the standard linearisation process using a truncation of the Taylor expansion of the non linear function.

4. APPROXIMATING NONLINEAR TRANSFORMATIONS OF PROBABILITY DISTRIBUTION THROUGH JULIER'S METHOD

The general problem of approximating nonlinear transformations of probability distributions can be stated as follows : given a r.v. $\boldsymbol{x} \in \Re^{n}$ with mean $\bar{\boldsymbol{x}}$ and covariance $\boldsymbol{P}_{xx}$, we would like to predict the mean $\bar{\boldsymbol{y}}$ and covariance $\boldsymbol{P}_{yy}$ of a r.v. $\boldsymbol{y} \in \Re^{m}$, where $\boldsymbol{y}$ is related to $\boldsymbol{x}$ by the non linear transformation $\boldsymbol{y} = \boldsymbol{f(x)}$. To solve this problem, Julier had an intuition that *with a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary non linear transformation*.

The procedure can be summarized as follows :

1. A set of weighted points $\{\chi_i\}_{i=1,\dots,2n+1}$ is chosen to approximate the r.v. $\boldsymbol{x}$:

$$\begin{aligned} \chi_0 &= \bar{\boldsymbol{x}} & W_0 &= \kappa/(n+\kappa) \\ \chi_l &= \bar{\boldsymbol{x}} + \left(\sqrt{(n+\kappa)\boldsymbol{P}_{xx}}\right)_l & W_l &= 1/2(n+\kappa) \\ \chi_{l+n} &= \bar{\boldsymbol{x}} - \left(\sqrt{(n+\kappa)\boldsymbol{P}_{xx}}\right)_l & W_{l+n} &= 1/2(n+\kappa)\ ,\ l = 1,\dots,n \end{aligned}$$

where $\kappa \in \Re$;

2. Each point is then transformed as $\mathcal{Y}_i = \boldsymbol{f}(\chi_i)$;

3. The mean $\bar{\boldsymbol{y}}$ is given by the weighted average of the transformed points : $\bar{\boldsymbol{y}} = \sum_{i=0}^{2n} W_i \mathcal{Y}_i$;

4. The predicted covariance $\boldsymbol{P}_{yy}$ is computed as : $\boldsymbol{P}_{yy} = \sum_{i=0}^{2n} W_i \{\mathcal{Y}_i - \bar{\boldsymbol{y}}\}\{\mathcal{Y}_i - \bar{\boldsymbol{y}}\}^T$.

In comparison with Monte Carlo techniques, the fundamental difference is that the samples are not drawn at random but according to a deterministic algorithm. A very small number of samples is then required to approximate the probability distribution. This method has many others advantages : owing to the parameter κ the scaling of the fourth and higher order moments can be influenced, the function ***f***, which may be implicit, is not approximated through a truncation of its series expansion, and the implementation is very easy since no evaluation of Jacobians is needed.

5. ESTIMATION OF SPATIAL UNCERTAINTY IN A *SLBL* SYSTEM

The Julier's method can be easily applied to estimate the spatial uncertainty of localization using a SLBL. The r.v. $\boldsymbol{x}$ is composed of all the noisy data required to estimate the position of the target. For an SLBL system with $\boldsymbol{N}$ buoys the components of r.v. $\boldsymbol{x}$ are:

- the Cartesian coordinates of the $\boldsymbol{N}$ buoys ($\boldsymbol{x_i}$ = longitude, $\boldsymbol{y_i}$ = latitude), $\boldsymbol{i = 1,..., N}$
- the propagation times $\boldsymbol{t_i}$, $\boldsymbol{i = 1,..., N}$
- the sound velocity $\boldsymbol{c}$.

The r.v. $\boldsymbol{y} = [x_T, y_T]^T$ denotes the position of the target where $\boldsymbol{x_T}$ and $\boldsymbol{y_T}$ represent the longitude and the latitude respectively.

$\boldsymbol{f}(.)$ is the nonlinear transformation that permits to obtain the position of the target :

$$\boldsymbol{f}(.) = \boldsymbol{Arg\ min}_y \sum_{i=1}^{N} \left(\boldsymbol{c} \times \boldsymbol{t_i} - \sqrt{(\boldsymbol{x_i} - \boldsymbol{x_T})^2 + (\boldsymbol{y_i} - \boldsymbol{y_T})^2} \right)^2$$

The method is then applied to estimate the mean and the covariance of $\boldsymbol{y}$. This result is depicted in a 95% confidence ellipse. To have a measure of the spatial uncertainty, we use the Frobenius norm of the covariance $\boldsymbol{P}_{yy}$.

6. ILLUSTRATIVE EXAMPLE

In this section we consider the experimental setup depicted in Fig. 5. An underwater mobile moves on a straight-line trajectory. The position of the target is to be tracked by a SLBL with three buoys ($\boldsymbol{B_1}$, $\boldsymbol{B_2}$, and $\boldsymbol{B_3}$). The goal of this experiment is to estimate the uncertainty on the current position during the mobile movement by using the second order statistics.

In order to show the efficiency of this method, sinusoidal model is used for the position errors of the buoys, as a model of the sea swell: $[\boldsymbol{x_i}, \boldsymbol{y_i}]^T = [\boldsymbol{x_{Bi}}, \boldsymbol{y_{Bi}}]^T + [\alpha_i \sin(\phi_i), \beta_i \sin(\psi_i)]^T$, where $\boldsymbol{x_{Bi}}, \boldsymbol{y_{Bi}}$ is the true position of the buoy $\boldsymbol{B_i}$ and ϕ_i and ψ_i are independent and uniformly distributed. Although this model is very far from a Gaussian, as shown by Fig. 3, it is seen on Fig.4 that the method performs well. For a three buoys-SLBL all the uncertain parameters have been represented by the r.v. $\boldsymbol{x} = [\boldsymbol{x_1}, \boldsymbol{y_1}, \boldsymbol{x_2}, \boldsymbol{y_2}, \boldsymbol{x_3}, \boldsymbol{y_3}, \boldsymbol{t_1}, \boldsymbol{t_2}, \boldsymbol{t_3}, \boldsymbol{c}]^T$. Note that only parameters $\boldsymbol{t_1}, \boldsymbol{t_2}, \boldsymbol{t_3}$ and $\boldsymbol{c}$ follow gaussian distribution. The covariance matrix is given by

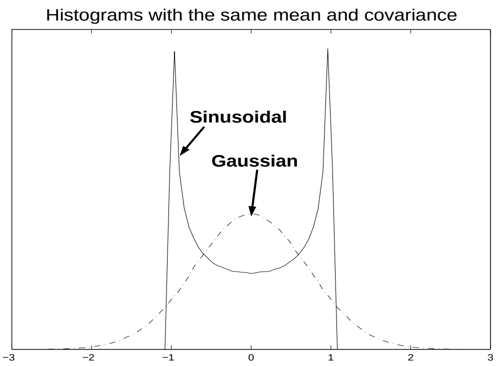
$$\boldsymbol{P_{xx}} = diag\left[\alpha_1^2/2, \beta_1^2/2, \alpha_2^2/2, \beta_2^2/2, \alpha_3^2/2, \beta_3^2/2, \sigma_{t1}^2, \sigma_{t2}^2, \sigma_{t3}^2, \sigma_c^2\right].$$

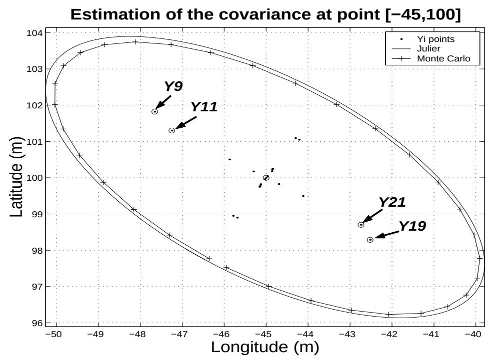
The initial position of the target is $[-100,120]^T$ and its final position is $[-120,40]^T$.

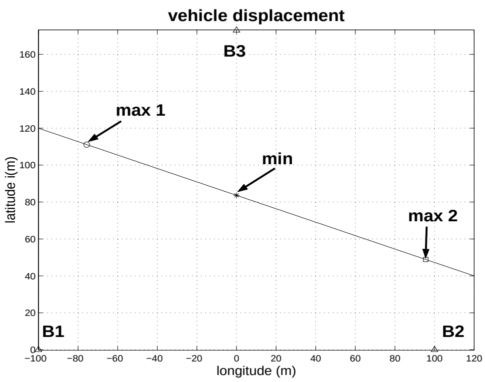
Fig. 4 represents the confidence ellipses calculated by the Monte Carlo method and the Julier's algorithm, at position $[-45,100]^T$. An accuracy of 3,3% is obtained using Julier's method ,with only 21 points ($n = 10$), against 10000 iterations for the Monte Carlo method.

The 21 samples $\{\mathcal{Y}_i\}$ of the resulting probability distribution are drawn on Fig. 4. It is interesting to note that the spatial uncertainty is essentially due to 4 points $\{\mathcal{Y}_9, \mathcal{Y}_{11}, \mathcal{Y}_{19}, \mathcal{Y}_{21}\}$, corresponding to the errors on second TOA $\boldsymbol{t_2}$ and the celerity. Hence, a great advantage of this method is the capacity to identify the influence of each parameter separately.

Fig. 6 shows the variations of the estimated value of the norm $\|\boldsymbol{P}_{yy}\|_F$ during the linear displacement of the mobile. It is clearly seen that the localization is more accurate near the middle of the SLBL [5].







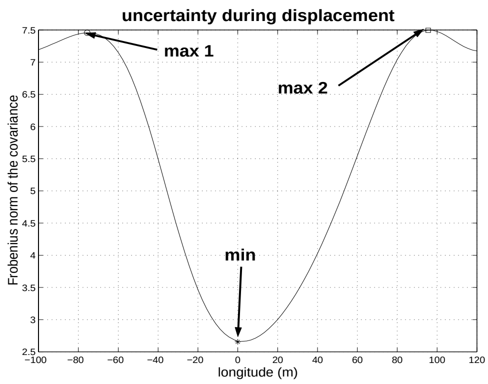


Fig.3: Sinusoidal and Gaussian distributions *Fig 4: 95% Confidence ellipses.*
Fig. 5: Simulation mobile trajectory *Fig. 6: Measure of localization uncertainty.*

7. CONCLUSIONS

The effects of inexact information have been studied in a SLBL acoustic positioning system. It has been emphasized that the spatial uncertainty is very sensitive to the layout of the buoys network and that this uncertainty does not necessarily reduces by adding buoys. The nonlinear transformations involved in the localization process have been handled owing to the Julier’s algorithm to approximate the resulting probability distribution. This method avoids the large computational cost usually encountered with Monte Carlo simulations and provides very accurate mean and covariance. Also, the ‘influence’ of each noisy parameter upon the spatial uncertainty can be clearly identified. These results should be useful to check in advance the performance of a SLBL configuration, given a particular positioning scenario.

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