Exercice 6

There is a large interest in tracking ballistic objects in their reentry phase into the atmosphere both in the field of defense and for safety against the reentry of some space debris (old satellites, for example).The tracking system typically relies on a radar which measures range and bearing. Due to the strong nonlinear forces acting on the object, a nonlinear stochastic filter is required for tracking. Many nonlinear Kalman filters have been investigated for addressing such problem over the past decades, including the classical Extended Kalman Filter (EKF) and more recently the Unscented Kalman Filter (UKF). In this study, the objective is to evaluate the performance of these two adaptive filters, considering some realistic target motion and radar measurement modeling[1].

Figure 1:

The forces acting on the target are gravity and drag. Some effects are ignored for modeling the kinematics, like centrifugal acceleration, Coriolis acceleration, wind, lift force, and spinning motion. Also, the flat Earth approximation is assumed.

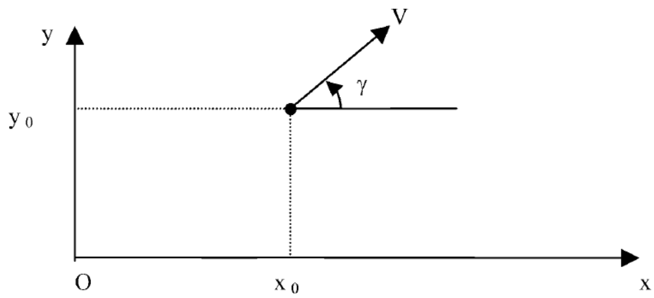

Figure 2: Coordinate reference system

The coordinate reference system illustrated in Figure 2 will be used for tracking the moving object, with abscissa x and ordinate y. At initial time t_0 the target with coordinate (x_0, y_0) is considered to be moving at speed v in a direction with angle γ with respect to the horizontal axis.

The following model with be considered for describing the ballistic object, with a state vector chosen as $\boldsymbol{s}_k = [x_k,\, \dot{x}_k,\, y_k,\, \dot{y}_k]^T$,

$$\boldsymbol{s}_{k+1} = \boldsymbol{\psi}_k(\boldsymbol{s}_k) + \boldsymbol{G}\begin{bmatrix} 0 \\ -g \end{bmatrix} + \boldsymbol{w}_k \tag{1}$$

where $\boldsymbol{\psi}_k(.)$ takes the form

$$\boldsymbol{\psi}_k(\boldsymbol{s}_k) = \boldsymbol{\Phi}\boldsymbol{s}_k + \boldsymbol{G}\boldsymbol{f}_k(\boldsymbol{s}_k)$$

with

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix}$$

[1]A. Farina, B. Ristic, and D. Benvenuti, " Tracking a ballistic target: comparison of several nonlinear filters, " *IEEE Trans. Aerosp. Electron. Syst.* , vol. 38, no. 3, pp. 854 – 867, Jul. 2002.

T denoting the sampling duration corresponding to radar measurement.

The drag force acts on the object in the opposite direction of the movement with an intensity expressed as $\frac{1}{2}(g/\beta)\rho v^2$,where g stands for the gravity acceleration constant, β is a ballistic coefficient and ρ corresponds to the air density. This one being dependent on the height as $\rho(y) = c_1 \exp(-c_2 y)$, with $\{c_1 = 1.227,\ c_2 = 1.093 \times 10^{-4}\}$ for $y < 9144$m and $\{c_1 = 1.754,\ c_2 = 1.49 \times 10^{-4}\}$ for $y \geq 9144$m. The drag force can be expressed as a function of the state variables as

$$\boldsymbol{f}_k(\boldsymbol{s}_k) = -0.5\frac{g}{\beta}\rho(\boldsymbol{s}_k[3])\left(\boldsymbol{s}_k^2[2] + \boldsymbol{s}_k^2[4]\right)\begin{bmatrix} \cos\left(\arctan\left(\frac{\boldsymbol{s}_k[4]}{\boldsymbol{s}_k[2]}\right)\right) \\ \sin\left(\arctan\left(\frac{\boldsymbol{s}_k[4]}{\boldsymbol{s}_k[2]}\right)\right) \end{bmatrix}$$

The above expression can be rewritten as

$$\boldsymbol{f}_k(\boldsymbol{s}_k) = -0.5\frac{g}{\beta}\rho(\boldsymbol{s}_k[3])\sqrt{\boldsymbol{s}_k^2[2] + \boldsymbol{s}_k^2[4]}\begin{bmatrix} \boldsymbol{s}_k[2] \\ \boldsymbol{s}_k[4] \end{bmatrix}$$

The process noise (1) can be considered as a white gaussian noise with zero mean and covariance

$$\boldsymbol{Q} = q\begin{bmatrix} \theta & \boldsymbol{0} \\ \boldsymbol{0} & \theta \end{bmatrix} \ , \ \theta = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}$$

where q is a parameter related to the noise intensity.

The radar, located at $\{x_R = 0,\ y_R = 0\}$, measures the location of the target with a range r and an elevation ϵ. The corresponding standard deviations are denoted as σ_r and σ_ϵ. The measures are transformed into Cartesian coordinates as $d = r\cos\epsilon$ and $h = r\sin\epsilon$, so as to get a linear measurement model:

$$\boldsymbol{z}_k = \boldsymbol{H}\boldsymbol{s}_k + \boldsymbol{v}_k \tag{2}$$

where

$$\boldsymbol{z}_k = [d_k,\ h_k]^T \ , \ \boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

The term $\boldsymbol{v}_k$ denotes the measurement noise in Cartesian coordinates, independent of the process model. It is assumed to be white Gaussian with zero mean and with a covariance $\boldsymbol{R}_k$ with entries

$$\begin{aligned} \sigma_d^2 &= \sigma_r^2\cos^2(\epsilon) + r^2\sigma_\epsilon^2\sin^2(\epsilon) \\ \sigma_h^2 &= \sigma_r^2\sin^2(\epsilon) + r^2\sigma_\epsilon^2\cos^2(\epsilon) \\ \sigma_{dh} &= (\sigma_r^2 - r^2\sigma_\epsilon^2)\sin(\epsilon)\cos(\epsilon) \end{aligned}$$

In this study, the following numerical values will be considered: $\beta = 40000$ kg.m^{-1}.s^{-2}, $q = 1$ m^2.s^{-3}, $T = 2$ s, $x_0 = 232$ km, $y_0 = 88$ km, $\gamma_0 = 190°$, $v_0 = 2290$ m/s, $\sigma_r = 100$ m, $\sigma_\epsilon = 0.017$ rad.

1. Simulate the target trajectory $\{\boldsymbol{s}_k\}$ from (1), by considering $\boldsymbol{w}_k = \boldsymbol{0}$ and give some illustration.
2. On some plots, show the variation of the speed and acceleration.
3. Generate noisy measurements[2] $\{\boldsymbol{z}_k\}$ from (2).
4. Linearize the process model (1) and express the Jacobian matrix F_k.
5. Implement an Extended Kalman Filter for tracking the target and evaluate its performance.
6. Implement an Unscented Kalman Filter for comparison purposes.

[2] As the coordinates d_k and h_k are impacted by correlated random noises, a Cholesky decomposition of the covariance matrix $\boldsymbol{R}_k$ is required to generate one random vector $\boldsymbol{v}_k$: $\boldsymbol{v}_k = \check{\boldsymbol{R}}_k^T.\boldsymbol{n}_k$, where $\check{\boldsymbol{R}}_k = \text{chol}(\boldsymbol{R}_k)$ and $\boldsymbol{n}_k \sim \mathcal{N}(0, \boldsymbol{I}.\sigma_n^2)$.